1 Geometric Series

When we add the terms of a geometric sequence together, we get a **geometric series**.

$2, 4, 8, 16, ...$ is a geometric sequence, but

$2 + 4 + 8 + 16 + ...$ is a geometric series.

2 Formulas for the Sum of a Geometric Series

In a geometric series of n terms,

$S_n = a + ar + ar^2 + ar^3 + ... + ar^(n-2) + ar^(n-1)$

$rS_n = ar + ar^2 + ar^3 + ar^4 + ... + ar^{n-1} + ar^n$

Subtract line 1 from line 2:

$rS_n - S_n = ar^n - a$

$rS_n(r-1) = a(r^n - 1)$

$$\boxed{S_n = \frac{a(r^n-1)}{r-1}, r \neq 1} \quad \text{or} \quad \boxed{S_n = \frac{t_1(r^n-1)}{r-1}, r \neq 1}$$

This formula connects a, r, n, and S_n.

If any three of these values are known, the fourth can be determined.

However, when the number of terms in the series (n) is not known, but t_n is known, *another* formula fo the sum can be formed, by replacing ar^{n-1} by t_n. The formula can now be written as:

$$\boxed{S_n = \frac{rt_n-a}{r-1}, r \neq 1} \quad \text{or} \quad \boxed{S_n = \frac{rt_n-t_1}{r-1}, r \neq 1}$$

If $r = 1$, the series is simply $a + a + a + ... = na$.

2.1 Example

Show how we arrive at $S_n = \frac{rt_n=a}{r-1}$ from $S_n = \frac{a(r^n-1)}{r-1}$.

2.2 Example

Determine the sum of the first fifteen terms of the sequence $-5, 10, -20, ...$

2.3 Example

Determine the sum $4 - 12 + 36 - ... - 8748$.

2.4 Example

The sum of a certain number of terms in the series $(-2) + 8 + (-32) + ...$ is -104858. What is the last term that would make this series add up to -104858?

2.5 Example

In a geometric sequence, the fifth term is 1024 and the common ratio is 4. Find the sum of the first seven terms of the sequence.

3 Determine the Number of Terms in a Geometric Series

The sum of n terms of the series $5 + 15 + 45 + ...$ is 16400. Determine the number of terms in the series by:

(a) solving an equation with a common base

(b) finding the intersection of two graphs

3.1 Example

A golf ball is dropped from the top of a building 100 m above a paved road. In each bounce, the ball reaches a vertical height that is $\frac{3}{4}$ the previous vertical height. Determine:

(a) the vertical height (to the nearest tenth of a metre) of the ball after seven bounces

(b) the total vertical distance (to the nearest tenth) travelled by the ball when it contacts the floor for the seventh time

(c) How many times does the ball need to bounce to travel approximately 675 m in vertical distance?

3.2 Example

Consider the geometric series defined by $S_n = 5(3^n - 1)$.

(a) Find the first four terms of the geometric series defined by $S_n = 5(3^n - 1)$.

(b) Find t_9 without using the formula $t_n = ar^{n-1}$.

Note: To find t_n in a series defined in terms of S_n, use the formula:

$$\boxed{t_n = S_n - S_{n-1}}$$

For example, $t_8 = S_8 - S_7$, or $t_4 = S_4 - S_3$.